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Applied Mathematics Letters 16 (2003) 1101–1104

Applied Mathematics Letters

www.elsevier.com/locate/aml

Local Nonobtuse Tetrahedral Refinements of a Cube

S. KOROTOV*
Department of Mathematical Information Technology
University of Jyväskylä
P.O. Box 35, FIN-40351 Jyväskylä, Finland
korotov@mit.jyu.fi

M. KŘÍŽEK†
Mathematical Institute, Academy of Sciences
Žitná 25, CZ-115 67 Prague 1, Czech Republic
krizek@math.cas.cz

(Received and accepted August 2002)

Communicated by F. Brezzi

Abstract—We propose an algorithm generating face-to-face partitions of a cube into nonobtuse tetrahedra that locally refine in a neighborhood of one of the vertices of the cube.

Keywords—Finite-element method, Nonobtuse tetrahedron, Path tetrahedron, Local refinement.

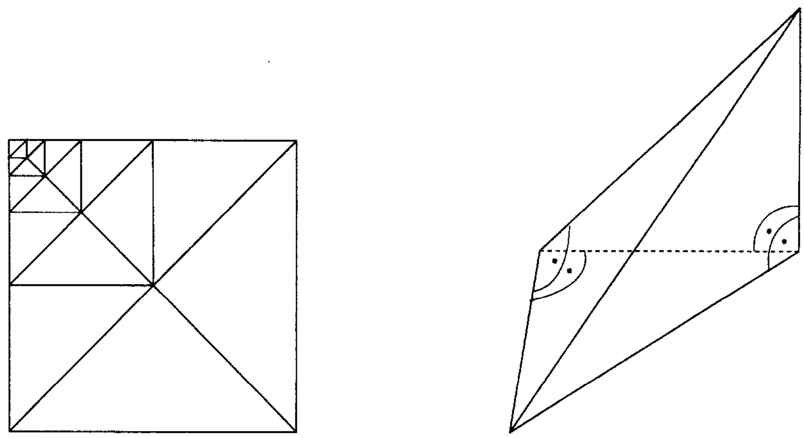
1. PRELIMINARIES

Nonobtuse simplicial elements play an important role in the finite-element analysis of boundary value problems, since they yield irreducible and diagonally dominant stiffness matrices and guarantee the validity of the discrete maximum principle (see [1]). In [2], we gave a global refinement algorithm which produces nonobtuse tetrahedra. However, local refinements of simplicial meshes are often necessary to handle vertex singularities of the solution. Figure 1a shows such a local nonobtuse refinement of a square into triangles near one of its vertices. In this note, we generalize this technique to a cube. The proposed algorithm can also be used to generate local nonobtuse tetrahedral face-to-face partitions of any rectangular domain composed of blocks (cf. Section 3).

Recall that a tetrahedron is said to be *nonobtuse*, if all six dihedral angles between faces are nonobtuse. Figure 1b shows a special type of the nonobtuse tetrahedron that has three mutually orthogonal edges, which do not intersect at the same vertex. Such tetrahedra are called *path tetrahedra* and will be used in the proposed refinements.

*The author was supported by Grant No. 49051 of the Academy of Finland.
†The author was supported by Grant No. A 1019201 of the Academy of Sciences of the Czech Republic.

0893-9659/03/$ - see front matter Typeset by $\mathcal{AMS}$-TeX
doi: 10.1016/S0893-9659(03)00150-2

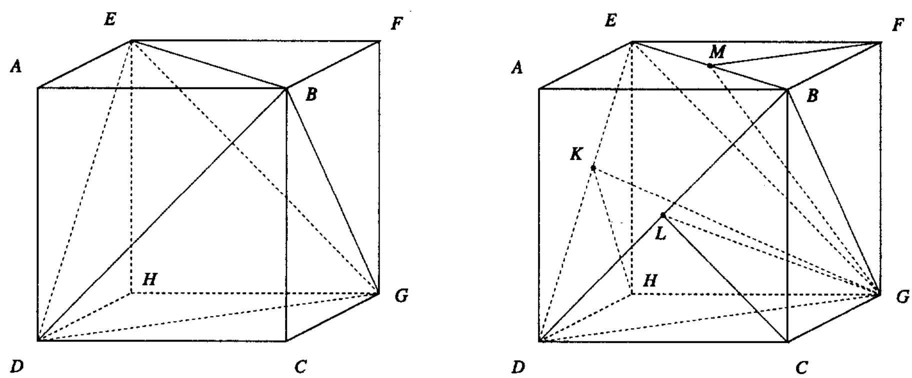


(a) (b)

Figure 1.

2. THE REFINEMENT ALGORITHM

Consider the cube $ABCDEFGH$ as sketched in Figure 2. Local refinements will be generated near the vertex A. The algorithm consists of five steps.



(a) (b)

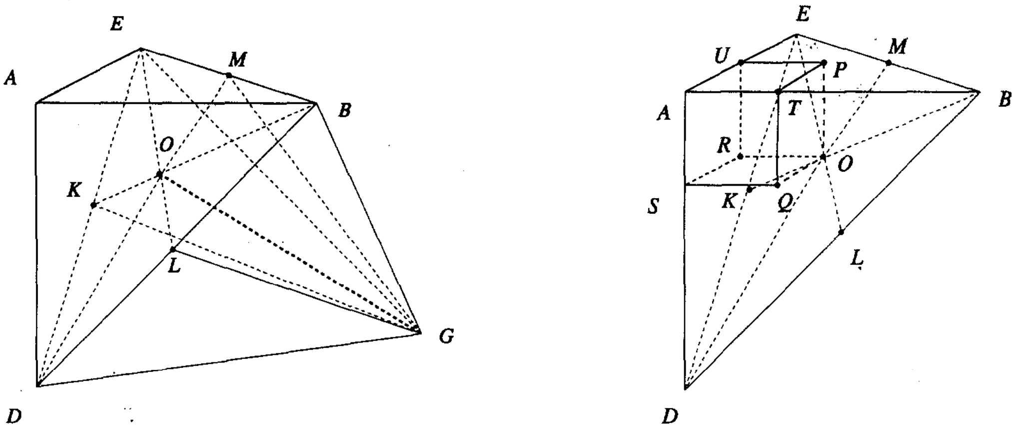
Figure 2.

(1) First, we decompose the cube into five nonobtuse tetrahedra as illustrated in Figure 2a.
(2) Each of the tetrahedra $CBDG$, $FBEG$, and $HDEG$ is bisected into two path tetrahedra as indicated in Figure 2b. For instance, the tetrahedron $CBDG$ is divided into $CLBG$ and $CLDG$, where L is the midpoint of the diagonal BD.
(3) Denote by O the center of the equilateral triangle BDE. The interior regular tetrahedron $BDEG$ is decomposed into six path tetrahedra

$$GOLB, \quad GOLD, \quad GOKD, \quad GOKE, \quad GOME, \quad GOMB,$$

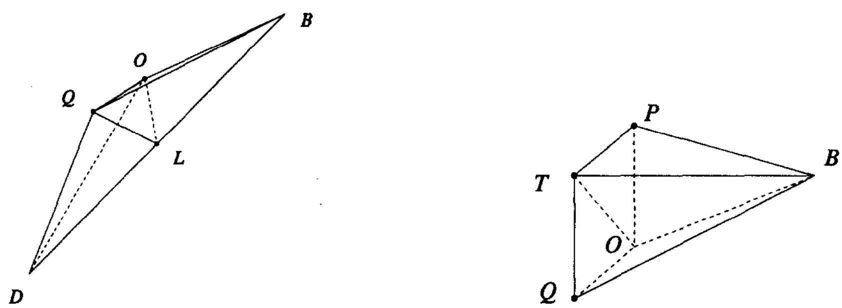
whose common edge is GO (see Figure 3a).
(4) It remains to decompose the last tetrahedron $ABDE$. Let the points P, Q, and R be the orthogonal projections of the point O onto the faces ABE, ABD, and ADE, respectively. This enables us to define the cube $ATQSUPOR$ as can be seen in Figure 3b. The remaining part can be decomposed into three congruent tetrahedra

$$BDQO, \quad BEPO, \quad DERO,$$



(a) (b)

Figure 3.



(a) (b)

Figure 4.

and three congruent pyramids

$$OQTPB, \quad OPURE, \quad OQSRD$$

with square faces. In Figures 4a and 4b, we observe how to bisect each of the above six elements into two path tetrahedra.

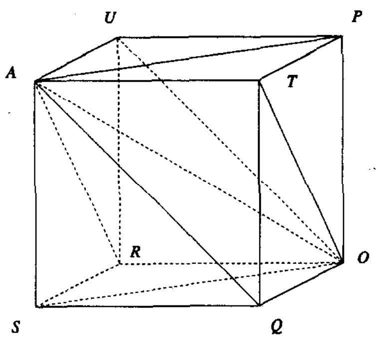
(5) The cube $ATQSUPOR$ can further be decomposed into five nonobtuse tetrahedra in a way similar to Figure 2a.

3. CONCLUDING REMARKS

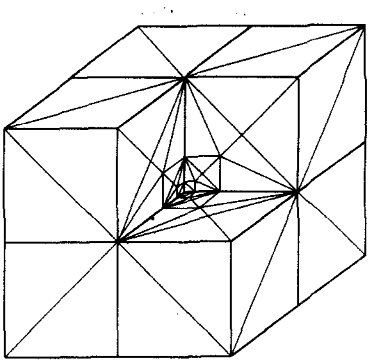
Repeating Steps (1)–(5) several times, we get a face-to-face partition into nonobtuse tetrahedra that locally refine in a neighborhood of the point A.

The final step (5) can be changed as follows. The smallest cube $ATQSUPOR$ can also be decomposed into six tetrahedra as depicted in Figure 5a. In this way, we obtain another local refinement, all of whose elements are only path tetrahedra.

If several congruent cubes meet at a singularity point (see, e.g., the so-called Fichera domain in Figure 5b), then we can apply the above algorithm to each of them so that the whole partition is face-to-face.



(a)



(b)

Figure 5.

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